ES120 Spring 2018 – Section 3 Notes

Matheus Fernandes

February 15, 2018

Problem 1:

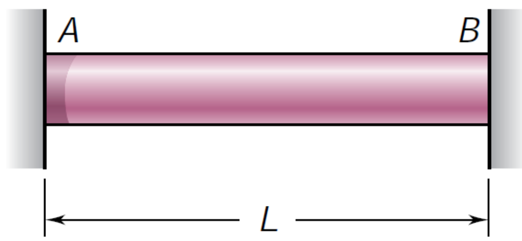

Figure 1

A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 45°F. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Knowing that $\alpha = 6.5 \times 10^{-6}$/°F, determine the stress in the bar (a) when the temperature is raised to 320 °F, (b) after the temperature has returned to 45°F.

Solution 1

Part A

First lets begin by analyzing the temperature required to cause yielding. It is always a great way to start when trying to break down the problem between it's linear part and the part where plastic deformation will begin to form. To do that, we need to recall the equation for deflection due to a load and balance that with the deflection caused by the thermal strain. Namely,

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) \tag{1}$$

We know that because they balance such that

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0 \tag{2}$$

Therefore, we can solve for $(\Delta T)_Y$ for which is the temperature required to begin yielding the material. If we solve for that from the above equation we get:

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36 \times 10^3}{(29 \times 10^6)(6.5 \times 10^{-6})} = 190.98°\text{F} \tag{3}$$

However, since we are raising the temperature to be $320°$F and $(320 - 45)°\text{F} = 275°\text{F} > 190.98°\text{F}$ we know that we are achieving the yield stress and thus the stress in the bar when the temperature is raised is

$$\boxed{\sigma_{max} = -\sigma_Y = -36\text{ksi}} \tag{4}$$

Part B

Now when we cool we the bar back by $\Delta T = 275$ we know that this will create a displacement that will be balanced as it was above in part A. So here we want to assume that there is no yielding so that we can superposed the yielding strain into it. This approach is exactly like the one we do for statically indeterminate problems. So we know that this fictitious displacement is

$$\delta' = \delta'_p + \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0 \tag{5}$$

Which we can rearrange to be:

$$\begin{aligned} \sigma' =& \frac{P'}{A} \\ =& -E\alpha(\Delta T)' \\ =& -(29\times 10^6)(6.5\times 10^{-6})(275) = -51.84\times 10^3\text{psi} \end{aligned} \tag{6}$$

So the residual stress is the stress from yielding, minus this superposed elastic stress that we computed which becomes

$$\boxed{\sigma_{res} = -\sigma_Y - \sigma' = -36\times 10^3 - (-51.8375\times 10^3) = 15.84\text{ksi}} \tag{7}$$

Problem 2:

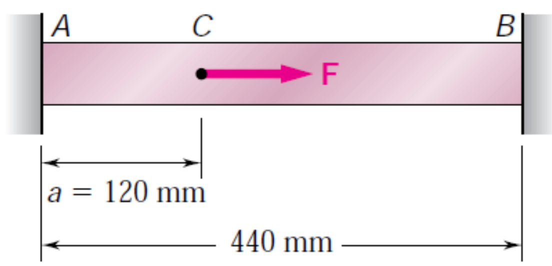

Figure 2

Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force F increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C, (b) the residual stress in the bar.

Solution 2

Part A

Fir this problem we are asked for the permanent deflection after the load has increased and decreased. As you can imagine, if we have stresses which are higher then yielding we have left the elastic regime and now have permanent deformation, meaning, we will not retreat to the original spot we started at. To begin, I like to get all most of my simple calculations out of the way, so let's compute the area we are interested in:

$$A = \pi r^2 = 1200\text{mm}^2 = 1200 \times 10^{-6}\text{m}^2 \tag{8}$$

Now let's compute the force required to yield portion AC:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6) = 300 \times 10^3\text{N} \tag{9}$$

For equilibrium to be true we can do a force balance such that

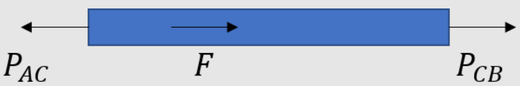

Figure 3

$$F + P_{CB} - P_{AC} = 0 \Rightarrow P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^{-3} = -220 \times 10^3 \text{ N} \tag{10}$$

Now using what we have left of the load, we can compute the stress in member BC and the deflection of point C such that

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220 \times 10^3)(0.44 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} - 0.293 \times 10^{-3} \text{ m} \tag{11}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-3}} = 183.33 \times 10^6 \text{ Pa} \tag{12}$$

Now when we unload we can use the compatibility fact that

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{BC}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{BC}}{EA} \tag{13}$$

$$P'_{AC}(\frac{L_{AC}}{EA} + \frac{LBC}{EA}) = \frac{FL_{BC}}{EA} \tag{14}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + LCB} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N} \tag{15}$$

Now we again know from force balance that

$$P'_{CB} = P'_{AC} - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N} \tag{16}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa} \tag{17}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-141.818 \times 10^3}{1200 \times 10^{-6}} = -118.18 \times 10^6 \text{ Pa} \tag{18}$$

$$\delta'_C = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^6)} = 0.189 \times 10^{-3} \text{ m} \tag{19}$$

So therefore, we know that the permanent displacement is the final yield displacement minus the elastic displacement, namely,

$$\boxed{\delta_{CP} = \delta_C - \delta'_C = 0.293^{-3} - 0.189 \times 10^{-3} = 0.1042 \times 10^{-3} \text{ m}} \tag{20}$$

Part B

Now for the stresses, we have already computed much of the work above and we just need to remove the elastic part from the yielding stresses and left over compressive stress from large member, namely,

$$\boxed{\begin{aligned} \sigma_{AC,res} &= \sigma_Y - \sigma'_{AC} = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \text{ MPa} \\ \sigma_{CB,res} &= \sigma_{CB} - \sigma'_{CB} = -183.3 \times^6 + 118.18 \times 10^6 = -65.2 \text{ MPa} \end{aligned}} \tag{21}$$