

# ES120 Spring 2018 – Section 6 Notes

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## Problem 1:

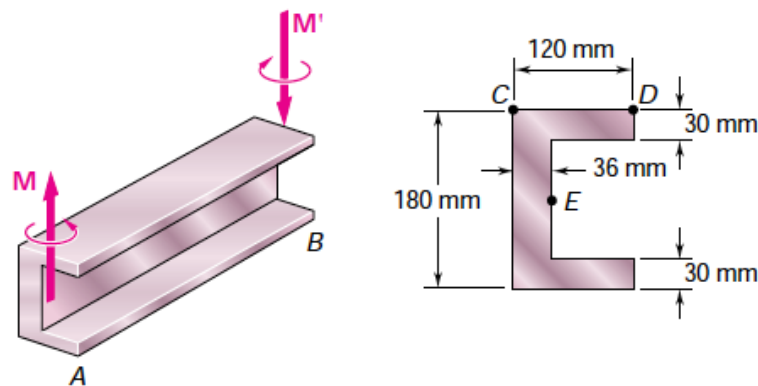


Figure 1

Two equal and opposite couples of magnitude  $M = 25$  [kN·m] are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress at (a) point C, (b) point D, (c) point E.

### Solution 1

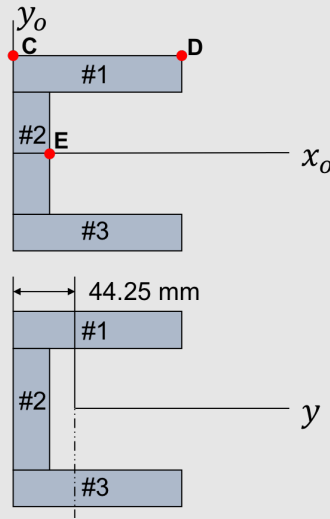
To do this problem, we need to identify what is the axis of the moment  $M$  applied to the beam. Since the axis of both moment arms are in the positive and negative  $y_o$  direction (according to the right hand rule), we need to first find what is the centroid of the structure within the plane of the cross section. To do so, we need to divide the cross section into three separate rectangles as shown fig. 2. Using the dimensions provided by the problem statement we can obtain measures for the area  $A$  of the different rectangles, and the distance  $\hat{x}_o$  in the direction  $x_o$  of the center of the rectangle. For these distances, we choose our reference axis to be  $y_o$ .

Now we can easily repeat this approach for all of the 3 rectangles in the cross section and formulate the first two columns of the table below. The last column is simply computed by multiplying the first two columns together.

	$A$ , [mm <sup>2</sup> ]	$\bar{x}_o$ , [mm]	$A\bar{x}_o$ , [mm <sup>3</sup> ]
#1	3600	60	$216 \times 10^3$
#2	4320	18	$77.76 \times 10^3$
#3	3600	60	$216 \times 10^3$
$\Sigma$	11,520		$509.76 \times 10^3$

Using this table we have all of the information we need to compute the centroid distance  $\bar{x}$  from the axis  $\bar{y}_o$  using the equation:

$$\begin{aligned} \bar{x}\Sigma A &= \Sigma A\bar{x}_o \\ \bar{x}(11520) &= 509.76 \times 10^3 \\ \bar{x} &= 44.25 \text{ [mm]} \end{aligned} \tag{1}$$



**Figure 2:** Diagram depicting different sections and centroid of cross-section.

Using the structure’s centroid information, we can now compute the distance  $d$  from a rectangles center to the centroid of the structure. We will get a different  $d$  for each of the rectangles

$$\begin{aligned} d_1 &= 60 - 44.25 = 15.75 \text{ [mm]} \\ d_2 &= 44.25 - 1.8 = 26.26 \text{ [mm]} \\ d_3 &= 60 - 44.25 = 15.75 \text{ [mm]} \end{aligned} \tag{2}$$

We’re ready to compute the second moment of inertia for the three different rectangles using the parallel axis theorem. **The parallel axis theorem states** that if the body is made to rotate instead about a new axis which is parallel to the first axis and displaced from it by a distance  $d$ , then the moment of inertia  $I$  with respect to the new axis is related to  $I_{cm}$  (center of mass) by

$$I = I_{cm} + Ad^2 \tag{3}$$

In other words, we must simply compute the moment of inertia of a given rectangle and offset the centroid by the distance between the two axis squared multiplied by the area. Always remember to define  $b$  and  $h$  according to the axis of rotation, which for this case is  $y_o$ . Let’s use this information to compute the moment of inertias for this problem

$$I_1 = I_3 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(120)^3 + (3600)(15.75)^2 = 5.2130 \times 10^6 \text{ [mm}^4\text{]} \tag{4}$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(120)(36)^3 + (4320)(26.25)^2 = 3.4433 \times 10^6 \text{ [mm}^4\text{]} \tag{5}$$

All of the work done so far gets us to the final goal of computing the moment of inertia of this slightly complex cross section. So the final step of this journey is to add these offset moment of inertias with respect to the centroid of the cross section together, namely

$$I = I_1 + I_2 + I_3 = 2I_1 + I_2 = 13.8694 \times 10^6 \text{ [m}^4\text{]} \tag{6}$$

Now for the different parts of the problem we must compute the distance of the points to our centroid axis  $y$ , namely:

$$\begin{aligned}y_1 &= -44.25 \text{ [mm]} \\y_2 &= 120 - 44.25 = 75.75 \text{ [mm]} \\y_3 &= 36 - 44.25 = -8.25 \text{ [mm]}\end{aligned}\tag{7}$$

At this point we have all of the information we need to compute the stresses at the different points requested for the different parts of the problem. So let's do this!

**Part (a)**

For Point C

$$\sigma_C = -\frac{My_C}{I} = -\frac{(25 \times 10^3)(-0.04425)}{13.8694 \times 10^{-6}} = 79.8 \times 10^6 \text{ [Pa]}\tag{8}$$

**Part (b)**

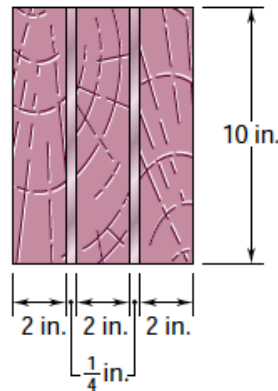
For Point D

$$\sigma_D = -\frac{My_D}{I} = -\frac{(25 \times 10^3)(0.07575)}{13.8694 \times 10^{-6}} = -136.5 \times 10^6 \text{ [Pa]}\tag{9}$$

**Part (c)**

For Point E

$$\sigma_E = -\frac{My_E}{I} = -\frac{(25 \times 10^3)(-0.00825)}{13.8694 \times 10^{-6}} = -14.87 \times 10^6 \text{ [Pa]}\tag{10}$$

**Problem 2:****Figure 3**

Three wooden beams and two steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity	$2 \times 10^6$ psi	$30 \times 10^6$ psi
Allowable stress	2000 psi	22,000 psi

**Solution 2**

We can easily identify that this problem is a composite problem. So now, we need to transform one material property into another using geometry. Therefore, let's pick wood as our reference material and scale the steel such that it is made of the same material but is instead thicker to compensate for its stiffness compared to the wood. To do that we introduce the ratio  $n$  with respect to the wood, namely,

$$n_w = 1 \quad \text{for wood} \quad (11)$$

$$n_s = \frac{E_s}{E_w} = \frac{30 \times 10^6}{2 \times 10^6} = 15 \quad \text{for steel} \quad (12)$$

Now we compute the properties of the geometric section, which is simply dependent on the geometry and does not need to be related to  $n$ s. This is a much simpler calculation than what we saw in the previous example because our axis is now the horizontal axis and the material is uniform w.r.t. the horizontal axis. Therefore, to compute the moment of inertia we simply compute:

$$I_s = \frac{1}{12} \left( \frac{1}{4} + \frac{1}{4} \right) (10)^3 = 41.6667 \text{ [in}^4\text{]} \quad (13)$$

$$I_w = \frac{1}{12} (2 + 2 + 2) (10)^3 = 500 \text{ [in}^4\text{]} \quad (14)$$

Now for the transformed section, we must only need to transform respective sections we found above to the scale we derived at the beginning of the problem. Therefore, that becomes,

$$I_{\text{trans}} = n_s I_s + n_w I_w = (15)(41.6667) + (1)(500) = 1125 \text{ [in}^4\text{]} \quad (15)$$

To obtain the largest permissible bending moment and relate that to the allowable stress we need to manipulate our equation below such that we get it as a function of the stress. The tricky part is to know that there is an  $n$  in the solution below. This makes sense, because we need to weigh the stress according to the ratio

of the stiffness. The only reason we can treat this difference in Young's moduli as a fraction of the geometry is because we're dealing with linear elasticity (and it's not non-linear).

$$|\sigma| = \left| \frac{nMy}{I} \right| \Rightarrow M = \left| \frac{\sigma_{\text{allow}} I}{ny} \right| \quad (16)$$

Now we need to evaluate this for both, the wood and the steel and see which one will fail first. The smaller value of the two will dictate the largest permissible bending moment of the member. Therefore,

$$M_{\text{wood}} = \frac{(2000)(1125)}{(1)(5)} = 450 \times 10^3 \text{ [lb} \cdot \text{in]} \quad (17)$$

$$M_{\text{steel}} = \frac{(22000)(1125)}{(15)(5)} = 330 \times 10^3 \text{ [lb} \cdot \text{in]} \quad (18)$$

Choosing the smaller value of the two, the steel will fail first at  $M = 330$  [kip · in]

 (19)