ES120 Spring 2018 – Section 8 Notes

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Problem 1:

The vertical shear is 25 kN in a beam having the cross section shown. Knowing that $d = 50$ mm, determine the shearing stress at (a) point a, (b) point b.

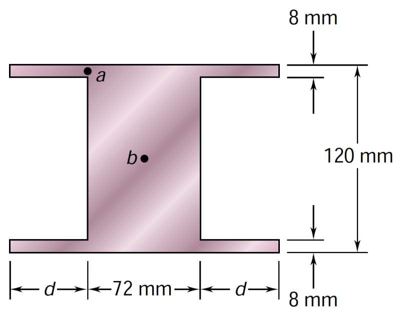

Figure 1

Solution 1

For this problem, let's first begin by computing the moment of inertia of the cross section using the outside parts and the inside part separately and using parallel axis theorem

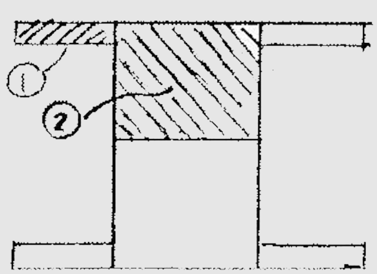

Figure 2

$$I_1 = \frac{1}{12}(50)(8)^3 + (50)(8)(56)^2 = 1.256 \times 10^6 \text{ mm}^4 \quad (1)$$

$$I_2 = \frac{1}{3}(72)(63)^3 = 5.184 \times 10^6 \text{ mm}^4 \quad (2)$$

Such that the total section's second moment of inertia becomes

$$I = 4I_1 + 2I_2 = 15.3933 \times 10^{-6}\ \text{m}^4 \tag{3}$$

Now we can compute the first moment with respect to the neutral axis Q

$$Q_1 = A_1\bar{y}_1 = (50)(8)(56) = 22.4 \times 10^3\ \text{mm}^3 \tag{4}$$

$$Q_2 = A_2\bar{y}_2 = (72)(60)(30) = 129.6 \times 10^3\ \text{mm}^3 \tag{5}$$

Part (a)

Now to determine the shearing stress at point a, we obtain the average shearing stress exerted on the element since it is located at the center of the element

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(25 \times 10^3)(22.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(8 \times 10-3)} \tag{6}$$

$$\boxed{\tau_a = 4.55\ \text{MPa}} \tag{7}$$

Part (b)

Similarly, to obtain the shear at point b, since it is at the center of the element, we only need to evaluate the average shearing stress exerted on the element. Remember, for these types of calculation we need to be working from the outside inward. So here we need to take account both of the first moments with respect to the neutral axis, namely

$$Q_b = 2Q_1 + Q_2 = 174.4 \times 10^{-6}\ \text{m}^3 \tag{8}$$

With this, we are ready to compute the average shear using the same equation as the previous problem, namely,

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(25 \times 10^3)(174.4 \times 10^{-6})}{(15.3933 \times 10^{-6})(72 \times 10^{-3})} \tag{9}$$

$$\boxed{\tau_b = 3.93\ \text{MPa}} \tag{10}$$

Problem 2:

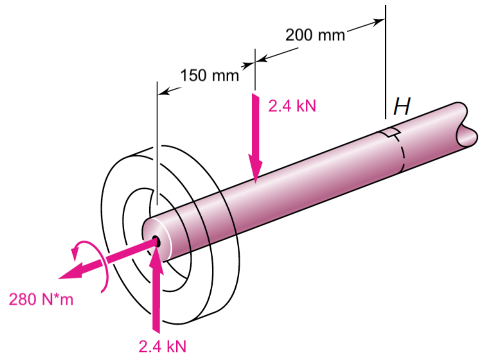

Figure 3

The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 1.25 in., determine (a) the principal planes and principal stresses at point H located on top of the axle, (b) the maximum shearing stress at the same point.

Solution 2

For this part, what we need to do is evaluate it the same way we did in the torsion problem, noting that H is located on the outer portion of the rod. So that the shear due to torsion becomes

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \tag{11}$$

Plugging in the values we get:

$$\tau = \frac{(2)(280)}{\pi(15)^3} = 52.8 \text{ kPa} \tag{12}$$

The bending second moment of inertia is

$$I = \frac{\pi}{4}c^4 = 39761 \text{ mm}^4 \tag{13}$$

The bending moment is

$$M = (150)(2400) = 360000 \text{ N} \cdot \text{m} \tag{14}$$

Thus the bending stress is

$$\sigma = -\frac{My}{I} = -\frac{(36 \times 10^4)(15)}{29761} = -135.8 \text{ MPa} \tag{15}$$

So now if we do a drawing of what is going on in term of the stress state at H we see the following:

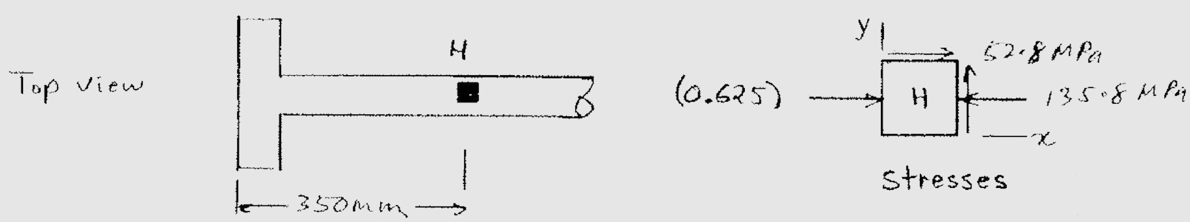

Figure 4

So, if we define a x and y axis we can decompose the stress states into the following

$$\sigma_x = -135.8 \text{ MPa} \tag{16}$$

$$\sigma_y = 0 \text{ MPa} \tag{17}$$

$$\tau_{xy} = 52.8 \text{ MPa} \tag{18}$$

So we van obtain the average stress as

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9 \text{ MPa} \tag{19}$$

The radius of the circle

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-67.9)^2 + (52.8)^2} = 86 \text{ MPa} \tag{20}$$

Part (a)

So to calculate the first principal stress we use

$$\sigma_1 = \sigma_{\text{ave}} + R = -67.9 + 86 = 18.1 \text{ MPa} \tag{21}$$

$$\sigma_2 = \sigma_{\text{ave}} - R = -67.9 - 86 = -153.9 \text{ MPa} \tag{22}$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-67.9} = -1.5552 \tag{23}$$

$$\boxed{\theta_p = -28.6^\circ \text{ and } 61.6^\circ} \tag{24}$$

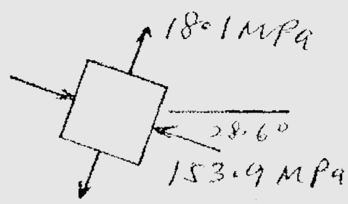

Figure 5

Part (b)

$$\boxed{\tau_{\text{max}} = R = 86 \text{ MPa}} \tag{25}$$

Problem 3:

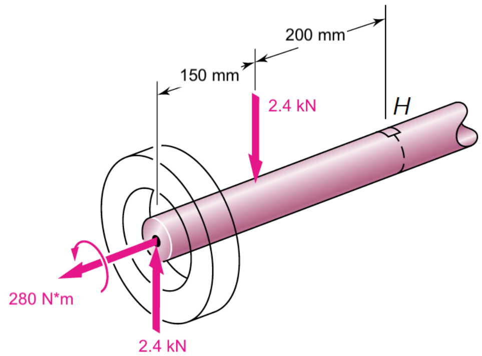

Figure 6

Solve the previous problem using Mohr's circle

Solution 3

Using the solutions from the previous problems, namely,

$$\sigma_x = -135.8 \text{ MPa} \tag{26}$$

$$\sigma_y = 0 \text{ MPa} \tag{27}$$

$$\tau_{xy} = 52.8 \text{ MPa} \tag{28}$$

$$\sigma\text{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -67.9 \text{ MPa} \tag{29}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-67.9)^2 + (52.8)^2} = 86 \text{ MPa} \tag{30}$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(52.8)}{-67.9} = -1.5552 \tag{31}$$

We can plot the the points from the previous problem, namely

$$X : (-135.8, -52.8); \quad Y : (0, 52.8); \quad \text{center} : (-67.9, 0) \tag{32}$$

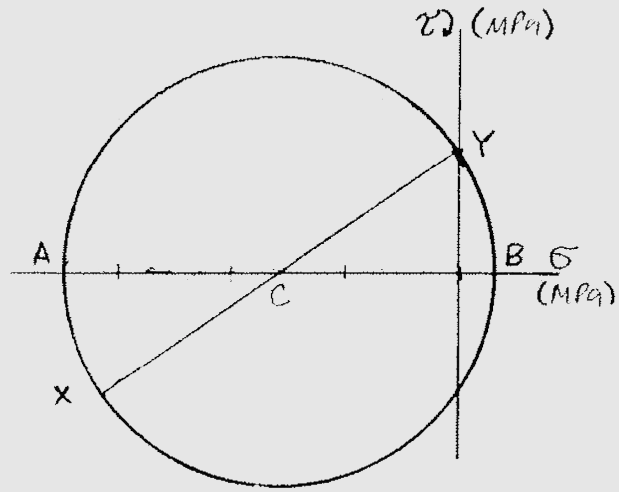

Figure 7

Part (a)

$$\boxed{\theta_p = -28.6° \text{ and } 61.6°} \quad (33)$$

Part (b)

$$\sigma_1 = \sigma_{\text{ave}} + R = -67.9 + 86 = 18.1 \text{ MPa} \quad (34)$$

$$\sigma_2 = \sigma_{\text{ave}} - R = -67.9 - 86 = -153.9 \text{ MPa} \quad (35)$$

$$\boxed{\tau_{\text{max}} = R = 86 \text{ MPa}} \quad (36)$$